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Soft Computing and Stochastic Optimization Approaches for Uncertain Design Parameters Determination of Post-Tensioned Composite Bridge

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Abstract: To achieve desired level of reliability in limit state design is generally not an easy task. Especially when probabilistic analysis including detailed description of uncertainties is utilized. In general, engineering design belongs to the category of inverse problems with the aim to determine selected design parameters. In the paper two alternative approaches are employed for finding design parameters of a single-span post-tensioned composite bridge. The first approach is based on utilization of artificial neural network in combination with small-sample simulation technique and genetic algorithms. The second approach considers inverse problem as reliability-based optimization task using small-sample double-loop method.

Keywords: Reliability-based design, inverse analysis, artificial neural network, double-loop optimization, post-tensioned bridge, reliability index, Latin hypercube sampling.

1. Introduction

When performing either reliability assessment or advanced engineering design, it is certainly essential to take uncertainties into account using a probabilistic analysis. Reliability assessment requires forward reliability methods for estimating the reliability (usually theoretical failure probability and/or reliability index are determined). On the other hand, the engineering design requires an inverse reliability approach to determine the design parameters to achieve desired target reliabilities.

In this paper two inverse reliability approaches are utilized. The first one is based on artificial neural network (ANN) in combination with small-sample simulation technique Latin hypercube sampling (LHS) and genetic algorithms (GA). General methodology of this inverse reliability analysis method was proposed by Lehký and Novák (2012).

The second method is double-loop reliability based optimization (RBO) approach. It aims at designing the system in a robust way by minimizing objective function under reliability constraints. It provides the means for determining the optimal solution of a certain objective function, while ensuring a predefined small probability of structural failure. Thus RBO method have to mix optimization algorithm together with reliability calculation. The approach known as "double-loop" consists in nesting the computation of the failure probability with respect to the current design within the optimization loop, see e.g. Dubourg et al. (2010).

2. Design parameters determination

2.1 Problem formulation

The aim of classical (forward) reliability analysis is the estimation of unreliability using a probability measure called the theoretical failure probability, defined as:

$$p_f = \mathrm{P}\left(Z \le 0\right) \tag{1}$$

where $Z = g(\mathbf{X})$ is a function of basic random variables $\mathbf{X} = X_1, X_2, \ldots, X_N$ called safety margin. The failure probability is calculated as a probabilistic integral:

$$p_f = \int_{D_f} f_{\mathbf{X}}\left(\mathbf{X}\right)\mathrm{d}\mathbf{X} \tag{2}$$

where the domain of integration of the joint probability distribution function (PDF) above is limited to the failure domain D_f where $g(\mathbf{X}) \leq 0$. The function $g(\mathbf{X})$, a computational model, is a function of random vector $\mathbf{X}$ (and also of other, deterministic quantities). Random vector $\mathbf{X}$ follows a joint PDF $f_X(\mathbf{X})$ and, in general, its marginal random variables can be statistically correlated.

The design of structure or its part to achieve the required reliability and durability is a typical example of the inverse problem. The aim is to find input design parameters (deterministic or associated with random variables) $\mathbf{d} \in \mathbf{X}$ which yield to the corresponding structural safety described by probability measures – failure probability p_f or reliability index β related to different limit states,

$$\mathbf{d} = f^{-1}(\mathbf{p_f}, \boldsymbol{\beta}) \tag{3}$$

Analytical solution of the inverse problem is usually possible only when using deterministic analysis and even just in simple cases. In other cases, often a trial-and-error procedure is carried out when an estimation of design parameters is performed (mostly based on empirical relationships and/or recommendations) and then the reliability of the system is assessed. Once we come to fully probabilistic analysis of structure an analytical solution or utilization of trial-and-error procedure is time-consuming and inefficient, or even impossible. Here, it seems necessary to use some advanced methods as it is described in the following sections.

2.2 Soft computing approach

The soft computing approach is based on the coupling of a small-sample stochastic simulation of Monte Carlo type and an ANN. Since finding analytical formulation of inverse function f^{-1} in (3) is possible only in extremely simple cases an ANN based surrogate model is utilized instead. Then the inverse problem (3) takes the form:

$$\mathbf{d} = f^{-1}_{\text{ANN}}(\mathbf{p_f}, \boldsymbol{\beta}) \tag{4}$$

where f^{-1}_{ANN} is an ANN approximation of the original inverse function.

ANN must be trained to solve the particular problem. Since the feed-forward type network is employed a “supervised” learning is used to adjust network parameters, i.e. a set of pairs (d_i, β_j) or (d_i, $p_{f,i}$), $d_i \in \mathbf{d}$, $\beta_j \in \boldsymbol{\beta}$, $p_{f,i} \in \mathbf{p}_f$ is introduced to the network with the aim to find a function $f^{-1}_{\text{ANN}} : \mathbf{d} \rightarrow \boldsymbol{\beta}$ or $f^{-1}_{\text{ANN}} : \mathbf{d} \rightarrow \mathbf{p}_f$ in the allowed class of functions that matches the examples. Here, LHS is used for the efficient preparation of training set. ANN training is an optimization task solved by GA in combination with gradient descent method. Once the ANN has been trained, it represents an approximation consequently utilized in the following way: To provide the best possible set of design parameters corresponding to prescribed reliability. It is done by introducing desired reliability measures to ANN as an input signal which is distributed through ANN structure to its output where optimal design parameters are obtained. For more detailed description of identification procedure see Lehký and Novák (2012).

2.3 Stochastic optimization approach

Another possibility how to find design parameters is to treat such inverse task as optimization problem, which is formulated as:

$$\begin{aligned}
&\text{find} - \mathbf{d}\\
&\min - f(\mathbf{d})\\
&\text{subject to}:\\
&p_f\left[g(\mathbf{d},\mathbf{X}) \leq 0\right] \leq p_0,\quad \mathbf{l} \leq \mathbf{d} \leq \mathbf{u}
\end{aligned} \tag{4}$$

with p_f the probability of constraint satisfaction. The limit state $g = 0$ separates the region of failure ($g \leq 0$) and safe region ($g > 0$) and is a function of the design variables $\mathbf{d}$ (and $\mathbf{l}$ and $\mathbf{u}$ are lower and upper bounds) and the uncertain variables $\mathbf{X}$, p_0 is the reliability level or performance requirement. The above inequality can be expressed by a failure probability multidimensional integral with the joint probability density function of probabilistic variables $\mathbf{X}$. Formulation based on reliability index instead of failure probability can be used.

The so called double-loop RBO approach has been chosen for identification of design parameters of analysed bridge in section 3. This approach splits calculations in two loops:

A) The outer loop represents the optimization part of the process. The simulation within the design space is performed in this cycle. For obtained design vectors of n-dimensional space $\mathbf{d}_i(d_1, d_2, \ldots, d_n)$ objective function values are calculated. The best realization is then selected based on these values. Consequently the best realization of random vector $\mathbf{d}_{i,\mathrm{best}}$ is compared with optimization constraints. These constraints are formulated as allowed interval of reliability index for selected limit state function. Calculations of reliability index for every randomly generated vector $\mathbf{d}_i$ takes place in the inner loop. Objective function itself has a functional value in the form of reliability index.

B) The inner loop is used to calculate reliability index (in presented example Cornell´s index) either for the need of checking of generated solutions, if they satisfy constraints, or to calculate the actual value of the objective function.

3. Post-tensioned Composite Bridge

3.1 Bridge description and computational model

A single-span post-tensioned composite bridge, crossing a single-track railway on the main road, is situated near the village Uherský Ostroh in the Czech Republic. The bridge was constructed in 1957. Based on the diagnostic survey from 2007, the bridge is made of twelve precast post-tensioned concrete MPD3 (outer) and MPD4 (intermediate) type girders, which were used from 1955 for construction of slab bridges up to a clear span of 18 m. Each of MPD girders was composed of six segments that are connected to each other by the transverse joints. See the bridge composition in Figures 1 and 2.

A computational model of the bridge was created in ATENA software (Červenka et al. 2012). For concrete a “3D NonLinear Cementitious 2” material model was used. Prestressing tendons and shear reinforcement were modelled as discrete and smeared reinforcement, respectively, by means of bilinear stress-strain diagram with hardening. The following load cases were modelled: dead load of the structure, longitudinal prestressing, secondary dead load and traffic load for the assessment of normal load-bearing capacity. Loading scheme related to normal loading class consists of a three-axle vehicle in every traffic line and a continuous load over the bridge width. For details see ČSN 73 6222 (2009). A computational model of the bridge, including the loading scheme described above is depicted in Figure 3. For an explanation a load caused by the front axle of the three-axle vehicle is replaced by the equivalent value of continuous load in particular traffic line.

Figure 1. A side view of analyzed bridge

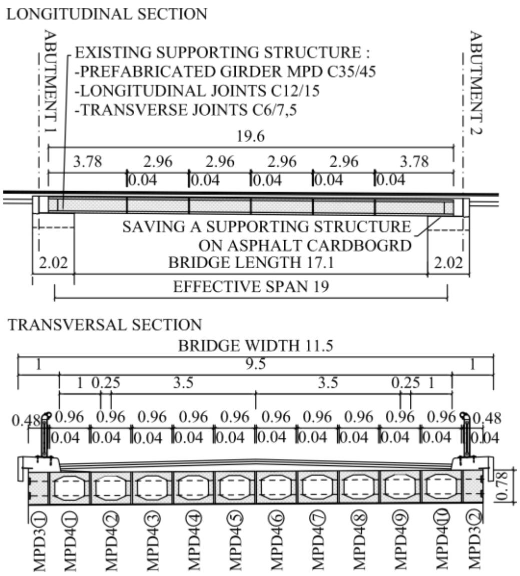

Figure 2. Longitudinal and transversal sections of analyzed bridge

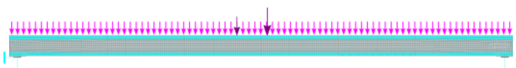

Figure 3. A computational model of the bridge, including the traffic load related to normal loading class

3.2 Stochastic model

For stochastic modeling, material properties of concrete and prestressing tendons were randomized. Stochastic parameters of random input variables were defined using FReET software (Novák et al. 2013) according to recommendations of JCSS (2013) and TP 224 (2010) and these were updated based on the material parameters testing according to diagnostic survey. Definitions of random input variables are summarized in Table 1. Alongside concrete material parameters, the dead load of the structure and the weight of road layers were randomized, see the concrete mass density and secondary dead load, respectively, in Table 1.

Table 1. Definition of input random variables of the model

Variable	PDF	Mean	CoV
Concrete of segments:			
Elastic modulus $E_{c,s}$ [GPa]	LN	37.20	0.10
Tensile strength $f_{t,s}$ [MPa]	WBM	3.301	0.15
Compressive strength $f_{c,s}$ [MPa]	LN	43.35	0.08
Fracture energy $G_{f,s}$ [N/m]	WBM	82.51	0.15
Mass density ρ_s [kN/m^3]	N	23.80	0.04
Concrete of transverse joints:			
Elastic modulus $E_{c,j}$ [GPa]	LN	26.81	0.15
Tensile strength $f_{t,j}$ [MPa]	WBM	1.913	0.35
Compressive strength $f_{c,j}$ [MPa]	Tri	19.13	0.23
Fracture energy $G_{f,j}$ [N/m]	WBM	47.82	0.25
Mass density ρ_j [kN/m^3]	N	23.80	0.04
Prestressing tendons:			
Elastic modulus E_p [GPa]	N	190.0	0.03
Yield strength $f_{y,p}$ [MPa]	N	1248	0.03
Ultimate strength $f_{u,p}$ [MPa]	N	1716	0.03
Prestress force P_1 [MN]	N	14.20	0.09
Prestress force P_2 [MN]	N	10.05	0.09
Prestress force P_3, P_4 [MN]	N	3.449	0.09
Other:			
Secondary dead load g_1 [kN/m]	N	65.55	0.05
Traffic load V_n [t]	Det	25	-

Note: Det – deterministic, N – Normal, LN – Lognormal, WBM – Weibull minimum, Tri – Triangular

Values of prestress forces were defined by their mean values with respect to short-term as well as long-term losses of initial prestress. Considering their substantial effect on global level of load-bearing capacity at the serviceability limit states, applied stochastic model was also defined fully in agreement with JCSS recommendations. Finally, traffic load was defined as deterministic.

The statistical correlation between material parameters of concrete of segments and transverse joints and prestressing tendons was also considered and imposed using a simulated annealing approach (Vořechovský and Novák 2009). Correlation matrices (see Fig. 4) were defined with respect to formerly performed tests and recommendations of JCSS.

a) Concrete of segments and transverse joints

	E_c	f_t	f_c	G_f	ρ
E_c	1	0	0.3	0	0
f_t	0	1	0.4	0.8	0
f_c	0.3	0.4	1	0	0
G_f	0	0.8	0	1	0

b) Pre-stressing tendons

	$f_{y,p}$	$f_{u,p}$	E_p	P_1-P_4
$f_{y,p}$	1	0.9	1	0
$f_{u,p}$	0.9	1	0	0
E_p	1	0	1	0
P_1-P_4	0	0	0	1

Figure 4. Correlation matrices of material parameters

3.3 Design parameters

According to diagnostic survey, the average value of concrete compressive strength of joints was 40.5 MPa but it was classified only as C6/7.5 strength class due to high variability in measurements probably caused by bridge spatial deterioration. This also brings uncertainty into the actual losses of prestress. Its value was roughly estimated according to code specifications as 17 % which correspond to the value of prestress force $P_1 = 14.20$ MN, including actual losses of prestress. Since the tensile strength of transverse joints and the bridge prestress has a significant effect on the bridge load-bearing capacity, mean values of both were considered as uncertain design parameters with the aim to find their critical values corresponding to desired reliability level and load-bearing capacity. Two limit states were taken into account – serviceability limit state of decompression (SLSD) and serviceability limit state of crack initiation (SLSC). The both limit states have implicit form – structural resistance is calculated using the nonlinear FE model, load action is considered as a deterministic variable placed according to the normal loading class scheme. Target reliability indices were considered as $\beta_1 = 0$ for SLSD, and $\beta_2 = 1.3$ for SLSC, respectively. According to diagnostic survey and needs of bridge administrator desired load-bearing capacity related to normal loading class was considered as 25 t.

Reliability analysis was carried out using LHS simulation method. Due to high computational demands of nonlinear model 32 simulations were used and Cornell's reliability indices for both limit states were calculated.

3.4 Determination using stochastic optimization approach

First, reliability based optimization approach was performed. Since SLSD is predominantly affected by level of bridge prestress, identification of its mean value for this limit state was performed first. Then, using known value of mean(P_1) also the second design parameter – mean value of tensile strength mean(f_t) – was taken into account and identified. Optimized design parameters along with corresponding reliability indices are presented in Table 2. Evolution of reliability indices during optimization is depicted in Figure 5.

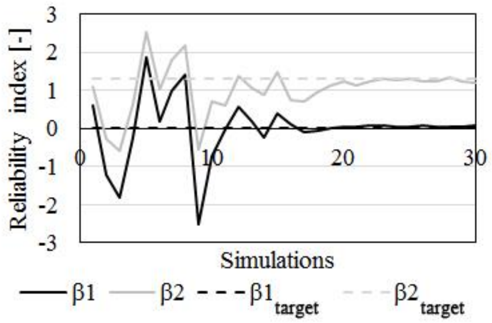

Figure 5. Evolution of reliability indices during optimization

Table 2. Resulting values of design parameters and corresponding reliability indices obtained by stochastic optimization approach

Design parameter	Value	β_1 ($\beta_{1,target}$)	β_2 ($\beta_{2,target}$)
mean(P_1) [%]	15.094	0.0638	1.3034
mean(f_t) [MPa]	2.88	(0)	(1.3)

3.5 Determination using soft computing approach

As an alternative to stochastic optimization method, the soft computing approach was employed. The ANN (see Fig. 6) consisted of one hidden layer having five nonlinear neurons (hyperbolic tangent transfer function) and an output layer having two output neurons (linear transfer function) which correspond to two design parameters – mean(P_1) and mean(f_t). The ANN has two inputs which correspond to two specified reliability indices, β_1 and β_2. The same set of random design parameter samples, generated using LHS method and corresponding reliability indices obtained from reliability analyses, which were prepared for stochastic optimization approach, were utilized here as the training set for ANN.

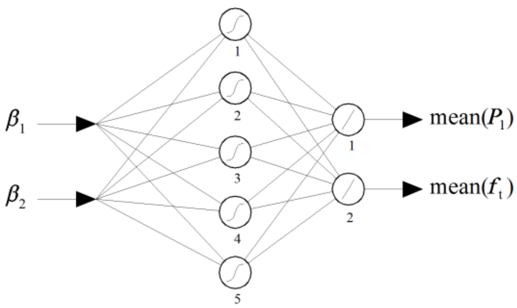

Figure 6. A schematic view of utilized ANN

Table 3. Resulting values of design parameters and corresponding reliability indices obtained by soft-computing approach

Design parameter	Value	β_1 ($\beta_{1,target}$)	β_2 ($\beta_{2,target}$)
mean(P_1) [%]	15.077	0.0734	1.2767
mean(f_t) [MPa]	3.04	(0)	(1.3)

The resulting design parameter values are summarized in Table 3. To validate results a stochastic analysis was carried out including determined design parameters and reliability indices were calculated; see the comparison with the target reliability indices in Table 3.

Results of both approaches show that the required mean values of concrete tensile strength in transverse joints correspond to compressive strength 37.7 MPa and 40.3 MPa respectively (calculated from tensile strength according to recommendations in *fib* Model Code 2010). These are smaller than original findings of diagnostic survey where the mean value of compressive strength was 40.5 MPa. Let's note that requirement for safety index $\beta_2 = 1.3$ in case of SLSC is relatively strict. For lower values of safety index, an even lower demand for concrete strength would be obtained.

Resulting requests to values of prestress forces are slightly stricter compared to those estimated according to code specifications where losses of prestress were considered as 17 % for infinite lifetime (9 % are immediate losses, coefficient of variation is 0.09). Identified mean value of prestress force indicates current loss of prestress equal to 12 %. From results we can conclude that requirement for normal load-bearing capacity $V_n = 25$ t is adequate for SLSC. In case of SLSD a more detailed investigation of losses of prestress and their variability would be necessary to confirm required load-bearing capacity for given safety.

4. Conclusions

Two efficient approaches for identification of selected design parameters of post-tensioned composite bridge were employed to ensure desired level of safety. Both approaches has led to more or less the same results with comparable computational demands. The most time-consuming part of identification is calculation of reliability when nonlinear FEM analyses are carried out. Both approaches are general and can be easily used for almost any inverse reliability problem. Results of inverse reliability analysis of post-tensioned composite bridge confirmed that bridge load-bearing capacity 25 tons related to normal loading class could be a realistic demand for required safety.

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