Shear resistance of prestressed girders: Probabilistic design

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Abstract: The paper describes a complex approach to probabilistic design of precast structural members made form advanced cementitious composites. First, a series of material, small scale component tests have been conducted in collaboration between two laboratories. Based on these tests identification of fracture-mechanical parameters (and their statistics) for two concrete mixtures used for the production of precast structural members was performed. Subsequently, studies have been performed on (a) full scale pre-stressed concrete roof elements (b) ten scaled laboratory tested elements. Mentioned experiments served as basis for deterministic nonlinear modelling of precast members and subsequent probabilistic evaluation of structural response variability. Final results may be utilized as thresholds for loading of produced structural elements and they aim to present probabilistic design as less conservative compared to classic partial safety factor based design and alternative ECOV method.

Keywords: Probabilistic analysis, nonlinear modelling, prestressed girders, reliability analysis, precast elements, shear resistance.

1 Introduction

The ultimate capacity of reinforced concrete beams subjected to combined shear and flexure is affected by many complex phenomena, such as existing multi-axial states of stresses, the anisotropy induced by the diagonal concrete cracking, the interaction between concrete and reinforcement (bond) and the brittleness of the failure mode. A large number of shear tests have been performed during the last decades, as summarized (Collins et al. 2008) in order to obtain valuable information about the shear transfer mechanisms. Currently, both in the US and in Europe, the shear design provisions/recommendations are undergoing a revision. Open questions concern, among others, the existence and magnitude of the size effect phenomena, the influence of reinforcement degree and the level of prestressing. Many different model formulations either mostly empirical (Bairan and Mari 2006; Ceresa et al. 2007) based on different physical mechanisms (Mari et al. 2015), or supported by fracture mechanics have been proposed.

The comprehensive experimental study of material parameters were performed first (Řoutil et al. 2014; Strauss et al. 2014; Zimmermann et al. 2014). It was understood from the early beginning that to develop very good numerical model for comparison with experiment of real structure would be impossible without the proper knowledge of fracture-mechanical parameters. Sampling and testing program has been performed by the Faculty of Civil Engineering, Brno University of Technology (BUT), Czech Republic and the Institute for Structural Engineering, University Natural Resources and Life Sciences (BOKU), Vienna in cooperation with the Austrian company Franz Oberndorfer GmbH & Co KG. The aim was to include in the database results for specified type of concretes the time of testing, the mean values, the standard deviations of the fracture mechanical concrete parameters and the most suitable mathematical model for probability distribution functions. In particular, three point bending (*3PB*) tests have been conducted on specimens made of C50/60 and C40/50 (Řoutil et al. 2014), with a central edge notch and analyzed via the effective crack approach and work-of-fracture method (Karihaloo 1995; Elices et al. 1992). By means of an advanced identification approach based on artificial neural network modeling (Novák and Lehký 2006; Lehký et al. 2014), the following three fundamental parameters of concrete were identified: modulus of elasticity E_c, tensile strength f_{ct}, and specific fracture energy G_f. The compressive strength of concrete f_c was measured by means of standard cubic compression tests.

The interest is primarily in the normal force-shear interaction of long-span TT concrete roof elements made of C50/60. Proof loading programs according to (Comité Euro-International 2012) for the characterization of the shear performance have been performed on three of these TT light concrete roof elements (Krug and Strauss 2013). Ten scaled T and rectangular shaped beams have been produced and tested (see Table 1).

Table 1: Testing program of the scaled concrete T and rectangular-shaped beams

Test series	Girder	Reinforcement layout		
1	T1: T30/150	S	V1	V2
	T2: R30/14	S		
2	T3: T45/14	S	V1	V2
	T6: R60/14			V2
3	T6: R60/14	V0	V1	

V0, S = non prestressed

V1 = 50% prestressed (compared to V2)

V2 = 100% prestressed

In particular, the layouts of aforementioned experiments have been designed in order to have a **(I)** similar stress condition in the shear fields for the pre-stressing P_o and body weight g_1, and **(II)** a similar fracture mode as for the afore mentioned TT roof elements. The ten laboratory beams were continuously prestressed with 449 MPa to 1105 MPA by four to eight strands St 1570/1770. The entire test program of scaled beams is shown in Table 1. The ten laboratory beams are characterized by a span length of 5.00 m, a height of 0.30/0.45/0.60 m, a web width of 0.14 m, the slab of the T shaped elements are characterized by a width of 1.50m and a thickness of 0.07 m.

The performance of the beams during their whole loading process was monitored by four monitoring systems, as described in Störzel et al. (2015), utilized together with the discussed identified stochastic parameters to introduce a comprehensive updating process of the nonlinear finite element models of the laboratory beams. Deterministic models of all performed experiments have been created in the next stage of the whole process. Brief description of basic features of models is presented in section 2. For more details about deterministic modelling see Slowik et al. (2015), Strauss el al. (2015), Novák et al. (2015).

This paper demonstrates the probabilistic evaluation of structural response performed for T shaped fully prestressed beam T30 150V2. Section 4 presents the first results obtained from reliability analysis performed with first set of 31 generated samples. Whole procedure of long-term comprehensive research is captured in Figure 1. Utilization of obtained response for extrapolation of test results to general geometrical properties of beams and to verify the analytical formulations of (Comité Euro-International 2012) for the shear resistance is scheduled as the further goal (Krug 2016).

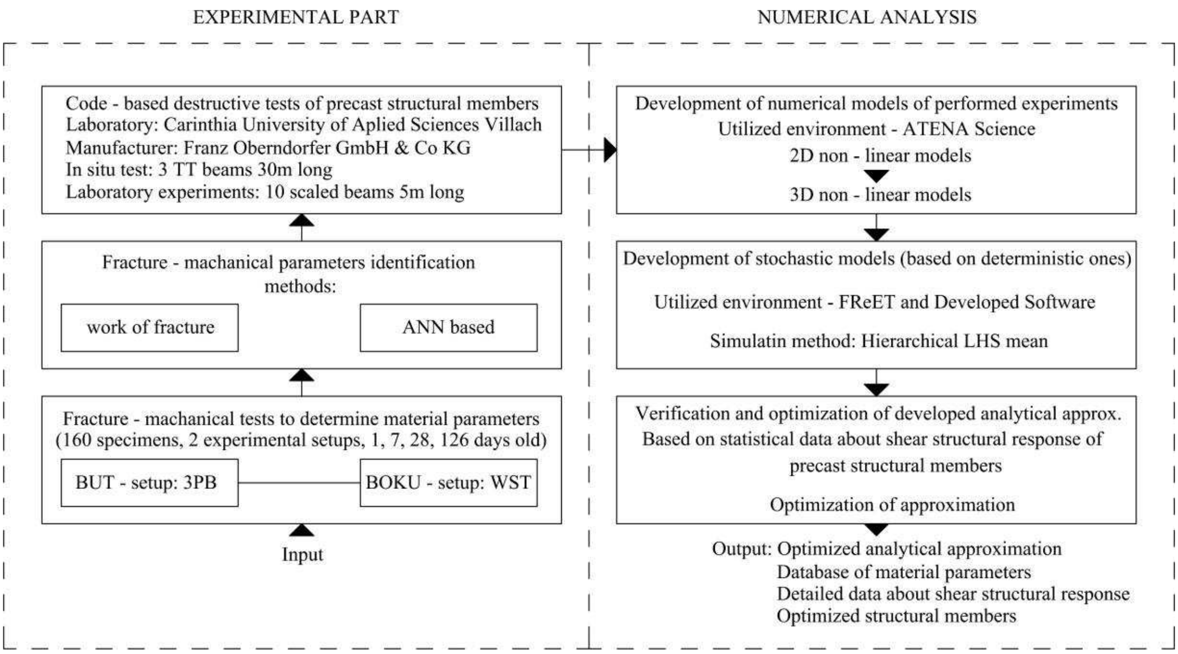

Figure 1: Scheme of the experimental and numerical studies associated with the research project on shear – normal force interactions of prestressed concrete girders

2 Deterministic computational model

Non-linear computational model of destructive test of scaled T – shaped pre-stressed girder was created using GID –ATENA Science software environment (Červenka et al. 2007). Geometry of beam and reinforcement was modelled exactly according to drawings provided by manufacturer.

Two linear supports were used for the model, since rollers were used during test. Regular hexagonal FE mesh composed of 16728 finite elements was generated in the program GID. The network has been condensed in the area of assumed shear failure. The FE mesh generated along with support conditions is shown in Figure 2.

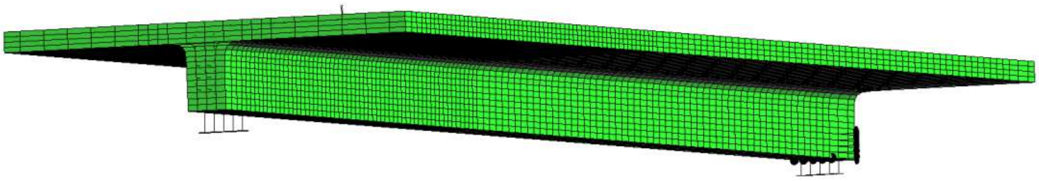

Figure 2: Finite element mesh and support conditions

Detailed model of reinforcement (including tendons) is visible in Figure 3. The steel reinforcement was modelled using 1D reinforcement material defined by strain vs. stress multilinear diagram. Prestressing tendons were also modelled using 1D reinforcement material, but working diagram of tendons was idealized as bilinear material with hardening.

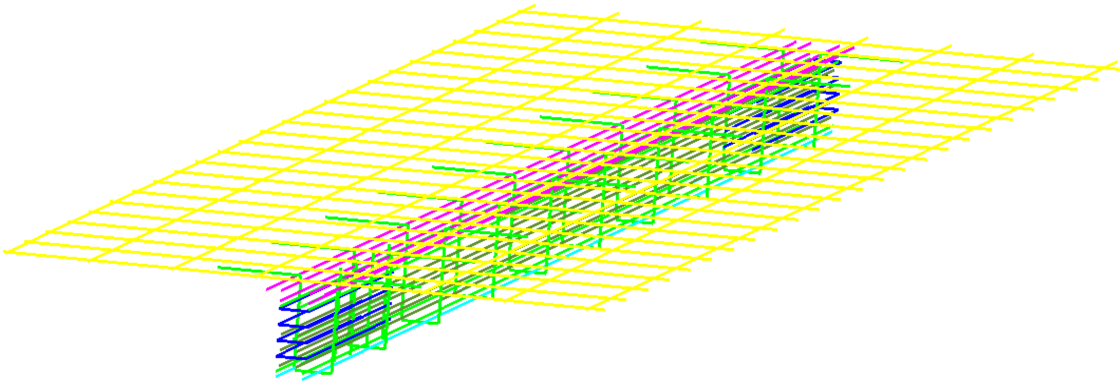

Figure 3: Reinforcement layout

For advanced modelling of concrete 3D Nonlinear Cementitious 2 material model was utilized. The CC3DnonLinCementitious2 material model is characterized by Rankine and Hordijk approaches for the description of the tension crack, the Collins + Vechio approach for the Aggregate Interlock effects, the Rankine fracturing model the characterization of the concrete cracking, the Hordijk approach for softening, and the Memetrey-Willam and Van Mier approaches for the plasticity behavior, see (Červenka et al. 2007).

Neglecting losses of prestressing could cause a significant inaccuracy of numerical model. Models without losses of prestressing have almost same ultimate capacity as models with consideration of losses. However, they underestimate final deflection of beam and resulting LD curve do not correspond to experimental one. Two different approaches have to be used simultaneously for applying prestressing losses realistically:

A) Reduction of prestressing itself

Prestressing is applied as initial strain for reinforcement line. This application will ensure that loss of prestressing due to elastic deformation of concrete is calculated explicitly. However, prestressing force was applied to concrete with age of only 14 hours during manufacturing process. This means that elastic modulus of concrete was different (approximately half, Grübl et al. 2001) than elastic modulus after 28 days of hardening. Initial strain for reinforcement line should be reduce by the difference between strain of concrete with age 14 hours and strain of concrete with age 28 days exposed to prestressing force. Second loss of prestressing which should be applied by reduction of initial strain for reinforcement line is loss due to relaxation of tendons.

B) Application of temperature load

Creep and shrinkage deformation leads to losses of prestressing. Test of girder was performed after 41 days of hardening. (Comité Euro-International 2012) was used for calculation of creep and shrinkage deformation. This deformation was recalculated to corresponding temperature load in order to influence stress state in reinforcement and tendons. This should decrease the rigidity of beam.

There are the following predominantly used monitoring systems for the updating process: The verification of the **(I) Load vs. vertical displacement (LD) diagrams** obtained in the associated loading point (see Figure 4).

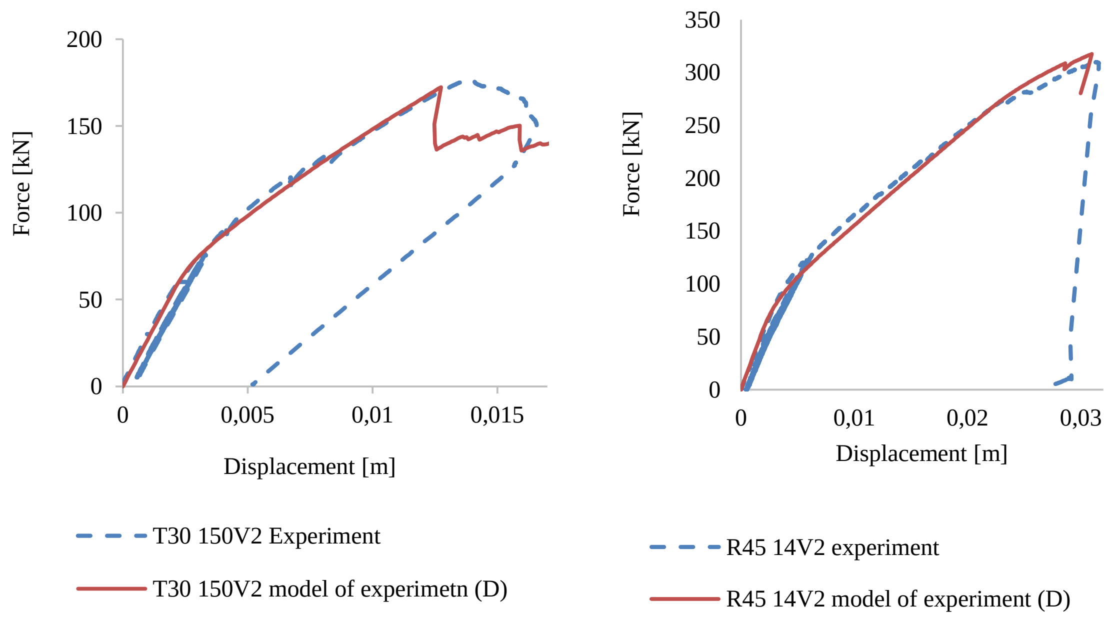

Figure 4 LD diagrams model vs. Experiments for beams T30 150V2 and R45 14V2

(II) Crack pattern development along an entire load cycle. **(III) strain gage** information attached on longitudinal and vertical oriented reinforcement bars in the shear field. Note, that model of experiment represent from statistical point of view model of one realization, as discussed in (Novák et al. 2015). More detailed information about deterministic models and theirs updating may be found in (Slowik et al. 2015; Strauss el al. 2015; Novák et al. 2015).

3 Stochastic computational model

The aim was to estimate statistical variability of shear capacity of prestressed girders and to propose probabilistically based design values or resistances. For the purpose of stochastic analysis the SARA GUI environment along with solver ATENA and reliability tool FReET (Novák et al. 2014) were utilized. Sensitivity analysis performed at the beginning of the whole process showed the most decisive/dominating parameters of nonlinear modelling. A set of 12 parameters was utilized for stochastic evaluation of structural response variability. Stochastic model of concrete was based on data presented in (Řoutil et al. 2014) obtained by ANN based identification (Novák and Lehký 2006; Lehký et al. 2013).

Identified material parameters of concrete were obtained from laboratory test under perfect condition – resulting in quite small variability of material parameters of concrete. In order to ensure realistic results it was necessary to introduce higher variability for obtained material parameters and to randomize also density of concrete mixture. Stochastic models of steel reinforcement (Bst 550B) and tendons (cables – ST 1570/1770) were based at recommendations of JCSS (2001) (variability) and information from manufacturer (mean value). Prestressing force was randomized according to the recommendation of JCSS (2001). For each realization also prestressing losses were calculated according to Comité Euro-International (2012). Nonlinear modelling of performed experiment showed some variability of prestressing loss around calculated value. Due to low number of experimental samples it was not possible to perform reliable estimation of corresponding uncertainties. Therefore it was decided to introduce uncertainty of calculated losses of prestressing with variability corresponding to recommendations in JCSS (2001).

The whole stochastic model of destructive test of girder T30 150V2 (see table 1 and section 2) is shown in Table 2, where E is Young´s modulus (E – concrete, E_s – steel reinf., E_t – tendons), f_t is tensile strenght, f_c is compressive strength, G_f is fracture energy, ρ is density of concrete mixture, f_{ys} is yield strength of steel reinforcement, f_{yt} is yield strength of tendons, I. is uncertainty for immediate losses of prestressing and L. T. is uncertainty for long term losses of prestressing. The utilized correlation matrix is presented in Table 3 – correlation coefficients are based on synthesis of experiments and expert opinion. In order to introduce required statistical correlations, the simulated annealing optimization method (Vořechovský and Novák 2009) is utilized within software FReET.

Table 2: Stochastic model of destructive test

Prameter	**Mean**	**COV [%]**	**PDF**	**Unit**	**Source**
Concrete (C50/60)					
E	34.8	20.6	WBL min (3 par)	[GPa]	(Řoutil et al. 2014)
f_t	3.9	20.6	GMB max EV I	[MPa]	(Řoutil et al. 2014)
f_c	-77	16.4	GMB min EV I	[MPa]	(Řoutil et al. 2014)
G_f	219.8	32.8	GMB max EV I	[J.m^{-2}]	(Řoutil et al. 2014)
ρ	0.0023	4	Normal	[kton/m^3]	(Řoutil et al. 2014)
Steel reinforcement (Bst 550B)					
E_s	200	2	Normal	[GPa]	(Ceresa et al. 2007)
f_{ys}	610	4	Normal	[MPa]	(Ceresa et al. 2007)

Tendons (Cables - St 1570/1770)					
E_t	195	2.5	Normal	[GPa]	(Ceresa et al. 2007)
f_{yt}	1387.88	2	Normal	[MPa]	(Ceresa et al. 2007)
Prestresing force					
P	0.0835	6	Normal	[MN]	(Ceresa et al. 2007)
Loss of prestresing (Uncertainties)					
I. L.	1	30	Lognormal	[-]	(Ceresa et al. 2007)
L. T. L.	1	30	Lognormal	[-]	(Ceresa et al. 2007)

Table 3: Utilized correlation matrix

	E	f_t	f_c	G_f	ρ	E_s	f_{ys}	E_t	f_{yt}	P	I.	L. T.
E	1	0.5	-0.8	0.5	0	0	0	0	0	0	0	0
f_t	0.5	1	-0.7	0.8	0	0	0	0	0	0	0	0
f_c	-0.8	-0.7	1	-0.6	0	0	0	0	0	0	0	0
G_f	0.5	0.8	-0.6	1	0	0	0	0	0	0	0	0
ρ	0	0	0	0	1	0	0	0	0	0	0	0
E_s	0	0	0	0	0	1	0.6	0	0	0	0	0
f_{ys}	0	0	0	0	0	0.6	1	0	0	0	0	0
E_t	0	0	0	0	0	0	0	1	0.6	0	0	0
f_{yt}	0	0	0	0	0	0	0	0.6	1	0	0	0
P	0	0	0	0	0	0	0	0	0	1	0	0
I.	0	0	0	0	0	0	0	0	0	0	1	0.5
L. T.	0	0	0	0	0	0	0	0	0	0	0.5	1

4 Design based on probabilistic assessment

4.1 Concept

Stochastic analysis of beam T30 150V2 described in this paper is the first from scheduled probabilistic assessments of laboratory tested girders. Due to enormous computational demands of such study it was decided to utilize efficient HSLHS (Vořechovský 2014) concept implemented in FReET software (Novák et al. 2014). This approach allows to extend the number of performed simulations after previously performed runs of LHS simulations. The aim is to perform analysis with lower number of generated samples at the beginning to fix possible errors and to use previous simulations in case that no errors will occur.

Statistics of response of above described girder using set of 11 simulations extended in second run of HSLHS method by another 20 simulations so the total amount of performed simulations is so far 31. The response of the assessed structural member was evaluated for two limit states **(I) Ultimate limit state** represented by critical value of force applied during experiment (peak of LD diagram). **(II) Serviceability limit state** represented by value of force in moment when first bending cracks occurs. Lognormal probability distribution is assumed for both limit states. Charts displayed in sections 4.2 and 4.3 shows comparison of fully probabilistic approach, classical calculation of response using partial safety factors and so called ECOV method (Holický 2006) to estimate design value. In case of fully probabilistic design, calculated percentile of shear resistance corresponds to probability 0.0012 according to recommendations in (EN 1990 2002) (based on separation of resistance and action of load variables). Figure 5 shows comparison of first 31 LD Curves for calculated simulations.

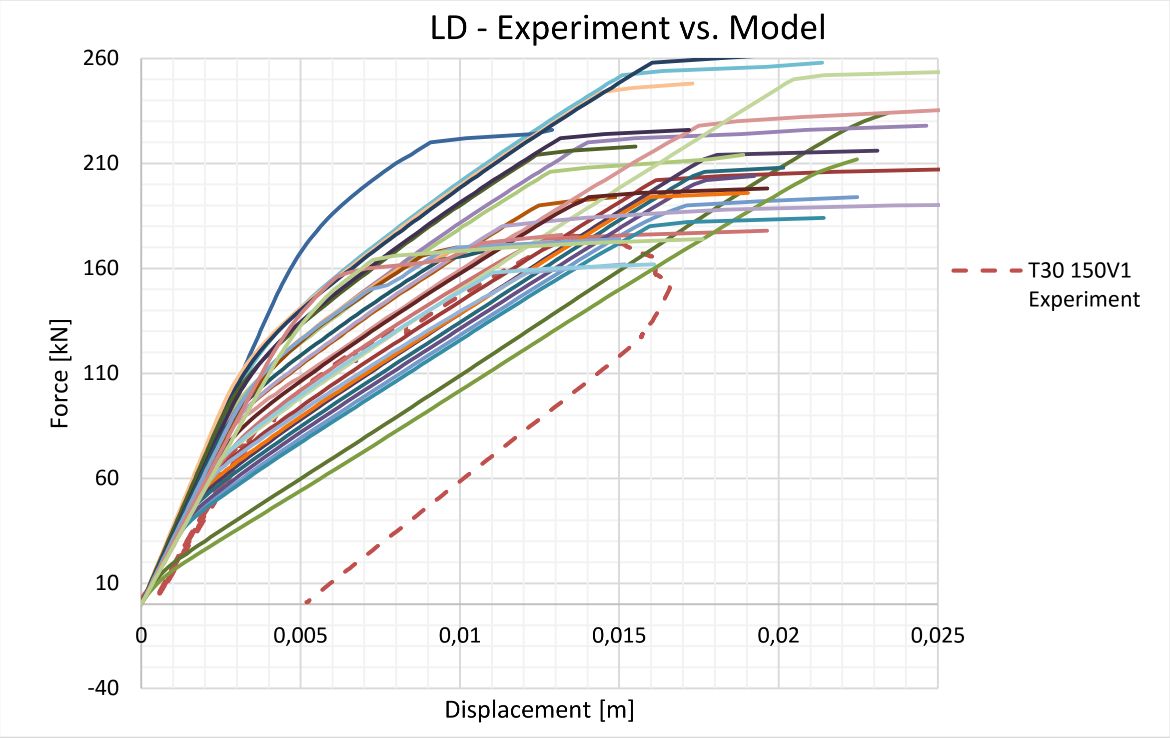

Figure 5: LD diagrams of first 31 calculated simulations

4.2 Ultimate limit state

The estimated probability distribution function (PDF) of shear resistance is shown in Figure 6. Structural response was considered to be lognormally distributed. Lognormal probability distribution function is utilized in order to demonstrate results and to compare calculated structural response with alternative approaches. More appropriate distribution function will be determined using curve fitting approach based on the complete set of 301 planed simulations. Calculated parameters of distribution function (mean value and standard deviation) are captured in figure along with design values (FP – fully probabilistic, PSF – partial safety factors, ECOV). Note, that all design alternatives are very close - ranging from 124 to 129 kN.

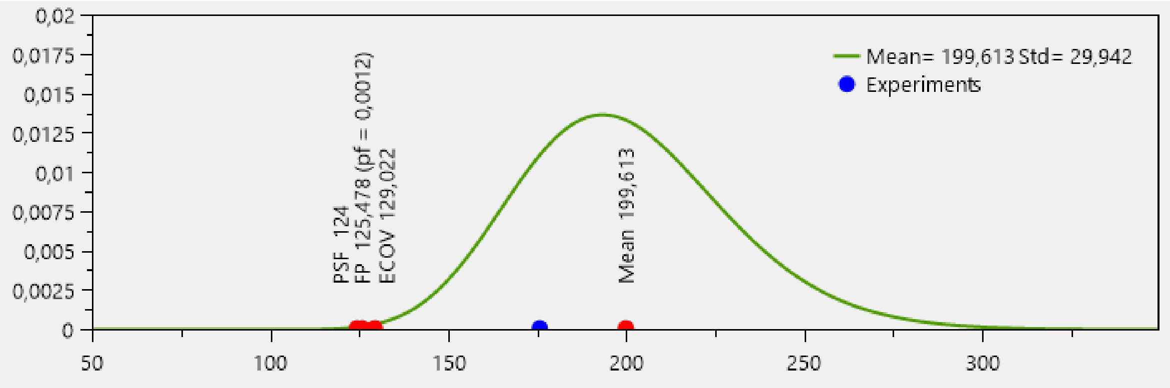

Figure 6: PDF of applied force at ultimate limit state

4.3 Serviceability limit state of crack initiation

Figure 7 captures PDF of applied force at point of first occurrence of bending cracks (serviceability limit state for fully prestressed components). Data are displayed, organized and compared in same manner as in Figure 6. ECOV technique value significantly deviates from other design alternatives here.

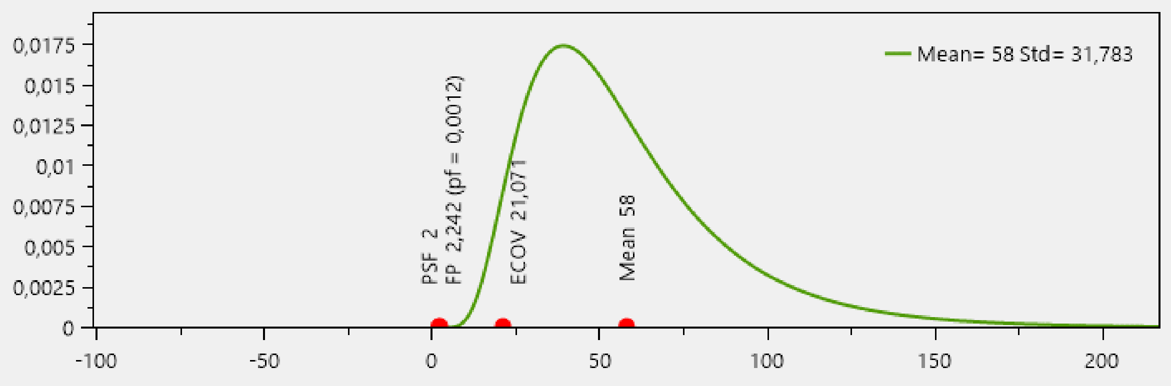

Figure 7: PDF of applied force at serviceability limit state

5 Conclusions

The paper describes integration/application of the modelling of nonlinearity and uncertainty to predict shear failure behaviour of prestressed concrete girders in the light of advanced design possibilities. The approach is complex, going from fracture-mechanical parameters determination and advanced deterministic 3D computational modelling of girders to stochastic modelling. The aim was to assess the variability of shear response and to present and verify alternative design procedures in comparison with fully probabilistic design. The paper describes this complex way, first results of stochastic analysis were presented for beam T30 150V2, more extensive studies will continue.

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